Tuning of robust distant downstream PI controllers for an irrigation canal pool: (I) Theory

X. Litrico, V. Fromion

Tuning of Robust Distant Downstream PI Controllers for an Irrigation Canal Pool: (I) Theory

X. Litrico*and V. Fromion†

Paper published in the *Journal of Irrigation and Drainage Engineering*, Vol. 132, No. 4, pp. 359-368, July/August 2006

Abstract

The paper proposes a new method to tune robust distant downstream PI controllers for an irrigation canal pool. The method emphasizes the role of gain and phase margins in the controller design, by linking the selection of these robustness indicators to the time domain specifications. This leads to link the frequency domain approach used by automatic control engineers to the time domain approach used by hydraulic engineers. The maximum error corresponding to an unpredicted perturbation is shown to be directly linked to the gain margin and the settling time to the phase margin of the controlled system. The tuning method gives analytical expressions for the controller parameters as function of physical parameters of the canal pool in order to satisfy desired performance requirements. The model is first expressed in terms of dimensionless variables, in order to get generic tuning formulas. The dimensionless PI coefficients are then expressed as functions of time-domain performance requirements. The PI tuning method is evaluated by simulation on a full nonlinear model for a canal pool taken from the ASCE Test Cases.

Introduction

The control of irrigation canals has been the subject of numerous scientific publications since the introduction of computers in the management of such large and complex systems (Malaterre and Baume, 1998). However, a systematic method to design simple and robust controllers for irrigation canals is still lacking. As stated by Wahlin and Clemmens (2002), "a rigorous method for tuning water level feedback controllers is desirable". The development of such a method is a challenging and critical issue, in the aim to improve water management of irrigation canals.

Such a method should enable a manager to easily tune controllers based on time domain requirements. Typical requirements could be formulated as follows (see Fig. 1):

1. For a given canal pool, the downstream water level should not deviate more than $\pm e_{\max}$ meters when a discharge perturbation of $\pm Q_p$ m^3/s occurs,

2. After such a perturbation has occurred, the time for the water level to reach its reference (at ± 10 %) should be at most t_{10} seconds,

3. These two design requirements should be fulfilled for various flow conditions (e.g. discharge, friction coefficient, discharge coefficient at gates, etc.),

4. Important implementation issues such as sampling-time specifications, or discharge/gate opening conversion should also be considered in the controller design. Computer-controlled systems work with a given sampling-time usually imposed by hardware specifications that imposes limitations

*Researcher, UMR G-EAU, Cemagref, B.P. 5095, 34196 Montpellier Cedex 5, France, Tel: +33 4 67 04 63 47. Fax: +33 4 67 63 57 95. e-mail: xavier.litrico@cemagref.fr

†Researcher, MIG, Mathematic, Computing Science and Genome, INRA, Domaine de Vilvert, F-78350 Jouy-en-Josas, France. E-mail: vincent.fromion@jouy.inra.fr

on the control. When the controller is designed using the discharge as control action variable, and the effective control action is the gate opening, one needs to convert the discharge into a gate opening. A realistic tuning method should take these issues into account.

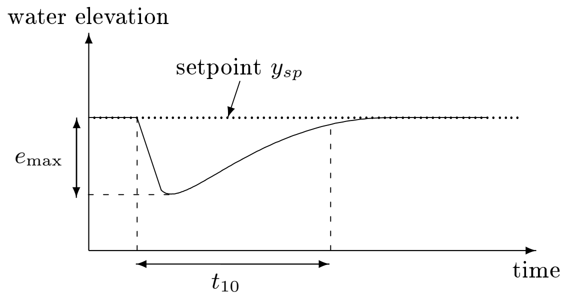

Figure 1: Definition of $e_{\max}$ and t_{10} for the downstream water elevation of a pool after a step perturbation

This paper deals with requirements 1 and 2, concerning the "nominal performance", for given hydraulic conditions. Based on this nominal performance design, the third and fourth requirements, concerning the "robust performance" and implementation issues can be handled in the same framework. These points are treated in a companion paper.
Let us consider the nominal performance problem (i.e., the design of a controller for a canal pool for nominal hydraulic conditions) in order to meet the two first requirements. We are strongly interested by simple controller design. Proportional Integral Derivative (PID) controllers are the simplest and most widespread controllers (Aström and Hägglund, 1995). For such controllers, the control input is proportional to the output error, the integral of the output error, and its derivative. However, the derivative action increases the complexity of the controller and makes the controller sensitive to sensor noise without drastically improving the performance in the case of delay systems Aström and Hägglund (1995). This is why the paper focuses on Proportional Integral (PI) controllers tuning.
Then, returning to the nominal performance problem, the first question that arises is: Does there exist a simple linear feedback controller that can fulfill the two first requirements? This is a feasibility problem. If yes, then what are the coefficients of a simple PI distant downstream controller that fulfills these requirements? These two questions have not been answered in the literature.
The approach classically used to tune PI controller for a canal pool is by trial and error (see Burt et al. (1998); Seatzu (1999); Weyer (2002); Wahlin and Clemmens (2002)) or by optimization (see Baume et al. (1999); Clemmens and Schuurmans (2004)). With a set of proportional and integral gains, one studies time-domain simulations to test how the controlled system reacts to a perturbation (i.e., discharge withdrawal). The controller parameters are then modified according to the simulations, in order to reduce the maximum error, or the settling time, or to minimize a given performance criteria. The difficulty is to trade-off between performance and robustness, since the controller should not only perform well for a given nominal situation (corresponding to a given discharge, roughness coefficient, etc.), but also for a canal pool with modified discharges and physical parameters. Such a trial-and-error approach is time-consuming and may not lead to a controller with an adequate performance in every case. It is also difficult with this approach to predict the stability of the controlled system for other types of perturbations.
Our objective is to derive a systematic and analytic tuning method of robust PI controllers for a canal pool. The proposed tuning method is based on gain and phase margins, which are classically used in control design to evaluate the robustness of a feedback loop with respect to gain and phase variations of the controlled system. These indicators also recover some part of the performance requirements. Indeed, we show that these robustness margins are closely related to the time-domain performance of the controlled canal pool. The maximum error corresponding to an unpredicted perturbation is shown to be directly linked to the gain margin and the settling time to the phase margin of the controlled

system. These margins are therefore very well suited as tuning parameters for a PI controller of a canal pool.
In order to obtain a generic tuning method, a simple linear model is used to represent the canal pool, the Integrator Delay (ID) model, whose parameters can be obtained directly from physical parameters of the canal pool (see Schuurmans et al. (1999a); Litrico and Fromion (2004b)). The canal pool ID model is first expressed in dimensionless values (i.e., the dimensionless time delay and integrator gain become 1). The analysis is carried out on this dimensionless system, keeping in mind the fact that the dimensional system will behave in a similar way as the dimensionless one with the appropriate transformations on time, discharges, and water levels. PI coefficients for specified gain and phase margins are then analytically derived from the ID model parameters. The interest is that the tuning formulas apply to any canal pool with given time-delay and integrator gain.
This paper focuses on the control of a single canal pool, and is illustrated by simulations on a full Saint-Venant nonlinear model of a canal pool taken from the ASCE test cases (Clemmens et al., 1998). We therefore assume that it is possible to design efficient decentralized controllers based on independently tuned controllers for each pool, and additional control elements added in order to minimize pool interactions (these elements are similar to the so-called "decouplers" of Schuurmans (1997)). This point is not developed in this paper or the companion paper, but will be the subject of future research.
Controllers usually incorporate feedback and feedforward, to deal with predicted perturbations. This paper only considers feedback control design, to deal with unpredicted perturbations.

Control Objectives

Irrigation canals are usually controlled using one of the two following classical control politics (Malaterre et al., 1998):

- distant downstream control, where the upstream control variables are manipulated to control a water level located at the downstream end of the pool,
- local upstream control, where the downstream control variables are manipulated to control a water level located just upstream.

The proposed PI controllers tuning method will be developed and illustrated for the distant downstream control case, but the method also applies for the local upstream control case, with minor modifications. Each pool is supposed to be controlled using the upstream discharge, and the controlled variable is the downstream water elevation. Throughout the paper, the deviation from upstream discharge is denoted u_1, the deviation from downstream discharge u_2, and the deviation from downstream water elevation y, while the outlet discharge is denoted p.

Controller Design Specifications

Control engineers usually handle the design specifications with two possible approaches:

- The most widespread approach is to design a controller for a given nominal situation, and then evaluate the controller over different situations. The typical procedure is to define a design model (corresponding e.g. to an average discharge and average values of physical parameters), design a controller for this model, and then evaluate the robustness a posteriori.
- The ideal approach, but the most difficult one, is to design a controller that guarantees a priori a certain minimum level of performance for a set of discharges and physical parameters such as friction coefficient, discharge coefficient, etc. This is the robust performance problem.

This paper shows how to handle the first approach by using the classical gain and phase margins. Gain and phase margins are directly linked to the time-domain performance of the system (i.e., the nominal performance) and also quantify the robustness of the system. This choice of tuning parameters will also facilitate the solution of the robust performance design problem, which is considered in the companion paper, together with implementation issues.

Linear Model for Control

In the following, $G_1(s)$ and $G_2(s)$ respectively denote the transfer functions from u_1 to y and from u_2 to y. s is the Laplace variable. The transfer function from the perturbation p to y is equal to G_2, since p acts as an additive perturbation on the downstream discharge u_2 (for simplicity, the unknown perturbation is assumed to be located at the downstream end of the canal pool). The canal pool is therefore represented by:

$$y = G_1(s)u_1 + G_2(s)(u_2 + p). \tag{1}$$

The ID model states that the transfer functions $G_1(s)$ and $G_2(s)$ can be approximated by (see Schuurmans et al. (1999a); Litrico and Fromion (2004b)):

$$G_1(s) = \frac{e^{-\tau_d s}}{A_d s} \tag{2}$$

$$G_2(s) = -\frac{1}{A_d s} \tag{3}$$

with τ_d the propagation delay of the pool (s) and A_d the backwater area (m^2).
As will be demonstrated below, this simple model is usually sufficient to capture the main dynamic properties of a canal pool for control design. For some cases, corresponding to short oscillating canals, it may be necessary to add a zero, leading an Integrator Delay Zero (IDZ) model. This point is detailed in Appendix I.

Application to Pool 4 of ASCE Test Canal 1

The paper is illustrated on pool 4 of ASCE Test Canal 1, taken from the ASCE Test Cases (Clemmens et al., 1998). The pool is 800 m long, with a bottom slope of 0.002 and a Manning's coefficient of 0.014 in tuned conditions. The pool has a trapezoidal geometry with bottom width 0.8 m and side slopes 1.5. The target downstream water level is 0.9 m.
For high flow conditions, the reference discharge is 1.4 m^3/s. The ID model used for PI controller design is obtained from Litrico and Fromion (2004b) leading to $A_d = 546$ m^2 and $\tau_d = 238$ s.
Fig. 2 depicts the Bode plot of the complete linearized Saint-Venant model (obtained following Litrico and Fromion (2004a)) vs the ID model.
In this diagram, the gain magnitude in dB and the phase in degrees of the system's transfer function are plotted vs the frequency (log scale).
Both models are very close up to 6.5×10^{-3} rad/s, which corresponds to the frequency where the phase of the system equals $-180°$.

Dimensionless ID Model

The system of Eqs. (1–3) can be normalized to dimensionless form by an adequate change of variables. Let Q_r be a reference discharge. In our case, the reference discharge is the maximum considered perturbation Q_p. Let us define $s^* = \tau_d s$ the dimensionless Laplace variable, $y_r = \tau_d Q_r / A_d$ the reference water level deviation, $y^* = y/y_r$ the dimensionless water level deviation, $u_1^* = u_1/Q_r$ the dimensionless upstream discharge deviation, $u_2^* = u_2/Q_r$ the dimensionless downstream discharge, and $p^* = p/Q_r$ the dimensionless water withdrawal. Then, the system of equation (1) is given by:

$$y^* = G_1^*(s^*)u_1^* + G_2^*(s^*)(u_2^* + p^*) \tag{4}$$

with $G_1^*(s^*) = e^{-s^*}/s^*$ and $G_2^*(s^*) = -1/s^*$. This dimensionless system has a time-delay equal to 1 and an integrator gain also equal to 1.

Dimensionless PI controller

Let the dimensionless controlled system be schematized as in Fig. 3.

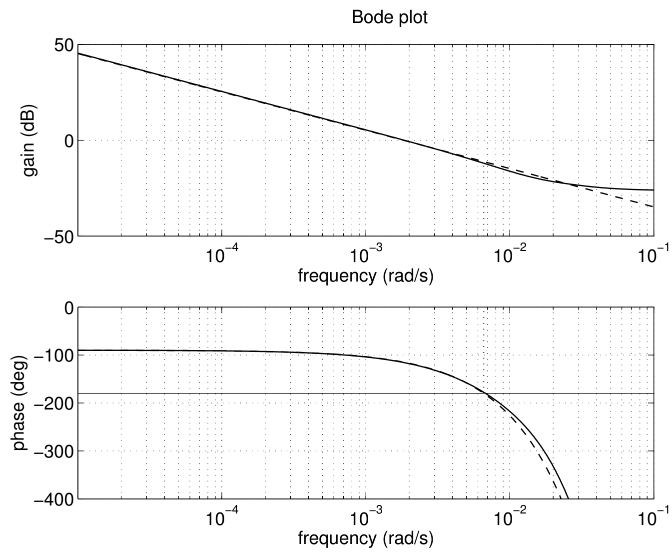

Figure 2: Bode plot of linearized Saint-Venant model (—) and ID model (– –) for pool 4 of ASCE Test Canal 1, high flow conditions.

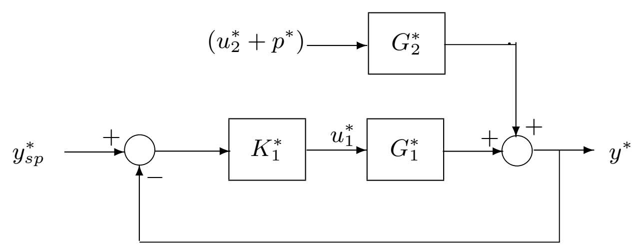

Figure 3: Schematic representation of distant downstream control of a dimensionless canal pool

The corresponding dimensionless PI controller will be denoted by:

$$K_1^*(s^*) = k_p^* \left(1 + \frac{1}{T_i^* s^*}\right)$$

with k_p^* the dimensionless proportional gain, T_i^* the dimensionless integral time.
This corresponds to a continuous controller where the control u_1^* is obtained by the equation:

$$u_1^*(t^*) = k_p^*(y_{sp}^*(t^*) - y^*(t^*)) + \frac{k_p^*}{T_i^*} \int_0^{t^*} (y_{sp}^*(v) - y^*(v))dv$$

where y_{sp}^* is the dimensionless set-point for the downstream water elevation.
Once a dimensionless PI controller with parameters (k_p^*, T_i^*) is designed on the dimensionless system, the dimensional PI controller parameters are obtained by:

$$k_p = k_p^* \frac{A_d}{\tau_d} \tag{5}$$

and

$$T_i = \tau_d T_i^*. \tag{6}$$

From Robustness Margins to Distant Downstream PI Tuning

Most classical textbook on automatic control emphasizes the importance of gain and phase margins for controller design and analysis (see e.g. Franklin et al. (2002)). These two key concepts are used by control engineers to analyze the performance and robustness of automatic controllers. The paper proposes a distant downstream PI controller design method for irrigation canal pools based on these two quantities. First, an analytical bi-univocal relation is derived between gain and phase margins and the PI controller parameters. Second, the time-domain performance of the controlled canal pool is shown to be directly related to the gain and phase margins.

Definition of Robustness Margins

Let us first recall the definition of the considered robustness margins. The absolute gain margin δg is the maximum multiplicative increase in the gain of the system such that the closed-loop remains stable. The gain margin is denoted ΔG when expressed in dB (i.e., $\Delta G = 20\log_{10}(\delta g)$). The phase margin $\Delta\Phi$ expressed in degrees is the maximum additive phase to the system such that the closed-loop remains stable.
The paper focuses on developing a method for a dimensionless canal pool using pre-specified robustness margins, because the chosen margins are independent of the dimensional variables. Indeed, the open-loop K_1G_1 is dimensionless, and it verifies the relation $K_1(s)G_1(s) = K_1^*(s^*)G_1^*(s^*)$. Therefore, if the dimensionless system has given gain and phase margins, so will the dimensional system.

Asymptotic Bode Diagram

For distant downstream control, the downstream water elevation is controlled using the upstream discharge. Then, the considered transfer function is $G_1^*(s^*)$, that relates the upstream discharge to the downstream water elevation. Let us denote by $K_1^*(s^*)$ the dimensionless distant downstream PI controller.
To study the frequency response of the controlled system, the open-loop transfer function $K_1^*(s^*)G_1^*(s^*)$ is evaluated on the imaginary axis $s^* = j\omega^*$, where ω^* is the dimensionless frequency and j the imaginary number such that $j^2 = -1$. This leads to:

$$|K_1^*(j\omega^*)G_1^*(j\omega^*)| = \frac{k_p^*}{\omega^*}\sqrt{1+\frac{1}{T_i^{*2}\omega^{*2}}} \tag{7}$$

$$\arg(K_1^*(j\omega^*)G_1^*(j\omega^*)) = -180 - \frac{180}{\pi}\omega^* + \frac{180}{\pi}\arctan(T_i^*\omega^*). \tag{8}$$

The asymptotic Bode plot gives a good estimate of the frequency response of a given system (Franklin et al., 2002). The asymptotic Bode diagram of $K_1^*G_1^*$ is depicted in Fig. 4. The gain of the open-loop decreases with a slope of -40 dB per decade for frequencies lower than $1/T_i^*$ (denoted by (-2)), then with a slope of -20 dB per decade for frequencies higher than $1/T_i^*$ (denoted by (-1)). The phase of the open-loop starts from $-180°$ at low frequencies, increases due to the integral term of the controller, and then decreases due to the delay of the system.
Two frequencies are of great interest when studying the open-loop of a controlled system: the crossover frequency where the gain of the open-loop equals 1, denoted ω_c^*, and the frequency where the phase of the open-loop equals $-180°$, denoted ω_{180}^*. These two frequencies are critical ones, since the phase margin is evaluated for $\omega^* = \omega_c^*$ and the gain margin is evaluated for $\omega^* = \omega_{180}^*$.
The frequency ω_{180}^* can be easily approximated if we assume that $\arctan(T_i^*\omega_{180}^*) \approx \pi/2$. Such an approximation is valid if $1/T_i^* \ll \omega_{180}^*$, typically if $T_i^* \gg 1$. Then, using Eq. (8), one gets:

$$\omega_{180}^* = \frac{\pi}{2}. \tag{9}$$

The frequency ω_c^* will depend on the chosen gain margin for the controlled system. In Fig. 4, increasing the gain margin will result in translating the line of slope -20 dB/decade towards the bottom, and therefore will result in lowering the crossover frequency ω_c^*. This qualitative behavior can be precisely quantified.

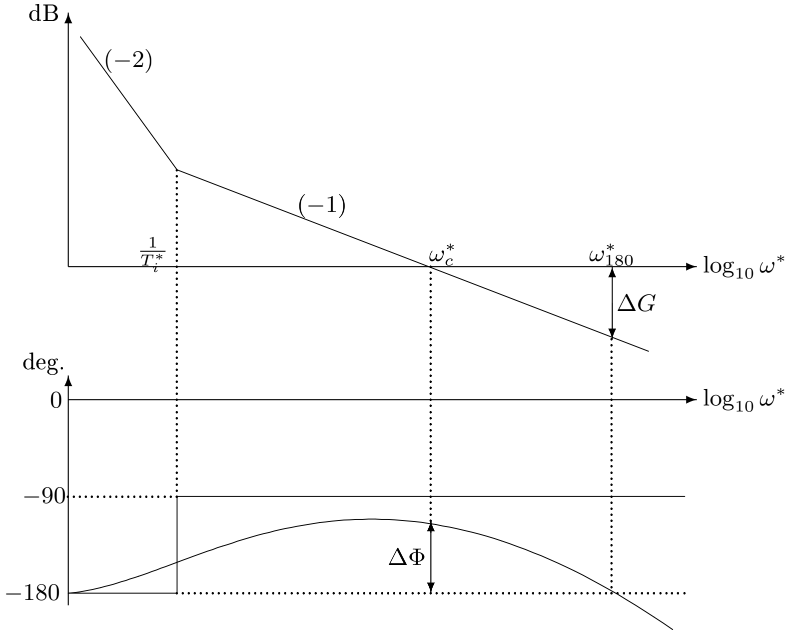

Figure 4: Schematic Bode plot of $K_1^* G_1^*$ for distant downstream control

PI Controller Tuning with Gain and Phase Margins

Let us specify a desired gain margin ΔG (in dB). As depicted in Fig. 4, between ω_c^* and ω_{180}^*, the Bode diagram is close to a line of slope -20 dB per decade. The relation between the crossover frequency ω_c^* and ω_{180}^* is then given by:

$$\omega_c^* = \omega_{180}^* 10^{-\frac{\Delta G}{20}}. \tag{10}$$

Therefore the crossover frequency is directly related to the gain margin of the controlled system.
Using the definition of the phase margin $\Delta\Phi$, the phase of the open-loop at the crossover frequency ω_c^* is given by:

$$\arg(K_1^*(j\omega_c^*)G_1^*(j\omega_c^*)) = -180 + \Delta\Phi.$$

Combining with Eq. (8), this leads to:

$$\Delta\Phi = \frac{180}{\pi}\left(\arctan(T_i^*\omega_c^*) - \omega_c^*\right). \tag{11}$$

A delay in the open-loop limits the achievable phase margin. Indeed, for a simple integrator system without a delay, the maximum achievable phase margin is 90°. For an integrator with delay, the maximum phase margin is obtained when $\arctan(T_i^*\omega_c^*) \approx \pi/2$, which gives an upper bound of the achievable phase margin in function of the desired gain margin:

$$\Delta\Phi_{\max} = 90 - \frac{180}{\pi}\omega_c^* = 90(1 - 10^{-\frac{\Delta G}{20}}). \tag{12}$$

Therefore, to tune the PI controller, one needs to specify a desired phase margin $\Delta\Phi$ smaller than the maximum achievable phase margin $\Delta\Phi_{\max}$. Then, T_i^* should be chosen as:

$$T_i^* = \frac{1}{\omega_c^*}\tan\left(\frac{\pi}{180}\Delta\Phi + \omega_c^*\right). \tag{13}$$

Table 1: Dimensionless PI coefficients (k_p^*, T_i^*) for different values of ΔG and $\Delta\Phi/\Delta\Phi_{\max}$

	ΔG				
$\Delta\Phi/\Delta\Phi_{\max}$	6 dB	8 dB	10 dB	12 dB	14 dB
0.5	$(0.728, 3.08)$	$(0.557, 3.13)$	$(0.427, 3.38)$	$(0.328, 3.80)$	$(0.254, 4.39)$
0.6	$(0.749, 3.92)$	$(0.581, 4.03)$	$(0.452, 4.39)$	$(0.352, 4.98)$	$(0.275, 5.80)$
0.7	$(0.766, 5.30)$	$(0.600, 5.49)$	$(0.471, 6.03)$	$(0.370, 6.88)$	$(0.291, 8.05)$
0.8	$(0.778, 8.04)$	$(0.614, 8.36)$	$(0.485, 9.23)$	$(0.384, 10.57)$	$(0.304, 12.42)$
0.9	$(0.785, 16.18)$	$(0.622, 16.86)$	$(0.494, 18.67)$	$(0.392, 21.45)$	$(0.311, 25.24)$

One can now compute the proportional gain of the controller. Since at the crossover frequency, one has $|G_1^*(j\omega_c^*)K_1^*(j\omega_c^*)| = 1$, Eq. (7) gives:

$$k_p^* = \frac{T_i^* \omega_c^{*2}}{\sqrt{1 + T_i^{*2}\omega_c^{*2}}}.$$

Combining this with Eq. (13), one gets:

$$k_p^* = \omega_c^* \sin\left(\frac{\pi}{180}\Delta\Phi + \omega_c^*\right). \tag{14}$$

Then, with the approximate model (2), one may compute the coefficients of a dimensionless distant downstream PI with desired gain and phase margins using Eqs. (9–10) and (13–14).
Table 1 gives the values of dimensionless PI coefficients for different couples $(\Delta G, \Delta\Phi/\Delta\Phi_{\max})$.

Delay Margin

The delay margin of a controlled system is the maximum additional delay in the loop such that the system remains stable. Indeed, a delay decreases the phase proportionally to the frequency. Therefore, since the phase margin gives the additional phase available before instability, the delay margin can be computed by the ratio of the phase margin in radians over the crossover frequency:

$$\Delta\tau^* = \frac{\pi}{180}\frac{\Delta\Phi}{\omega_c^*}.$$

Since there is a maximum available phase margin, the maximum dimensionless delay margin of the controlled system is given by:

$$\Delta\tau_{\max}^* = \frac{\pi}{180}\frac{\Delta\Phi_{\max}}{\omega_c^*}. \tag{15}$$

Collecting Eqs. (10), (12), and (15) leads to the following result: to one gain margin ΔG corresponds a dimensionless crossover frequency ω_c^*, a maximum phase margin $\Delta\Phi_{\max}$, and a maximum dimensionless delay margin $\Delta\tau_{\max}^*$ (see Table 2).

Table 2: Dimensionless crossover frequency ω_c^*, maximum phase margin $\Delta\Phi_{\max}$ and maximum dimensionless delay margin $\Delta\tau_{\max}^*$ for different gain margins ΔG

ΔG (dB)	4	6	8	10	12	14	16
ω_c^*	0.991	0.787	0.625	0.497	0.395	0.313	0.249
$\Delta\Phi_{\max}$	33°	45°	54°	61°	67°	72°	76°
$\Delta\tau_{\max}^*$	0.58	1	1.51	2.16	2.98	4.01	5.30

The crossover frequency corresponds to the system bandwidth. This frequency is a measure of the time-domain performance of the controlled canal pool, since perturbations occurring at a frequency higher than ω_c^*/τ_d rad/s will not be efficiently rejected by the controller. For example, with $\Delta G = 10$ dB, the

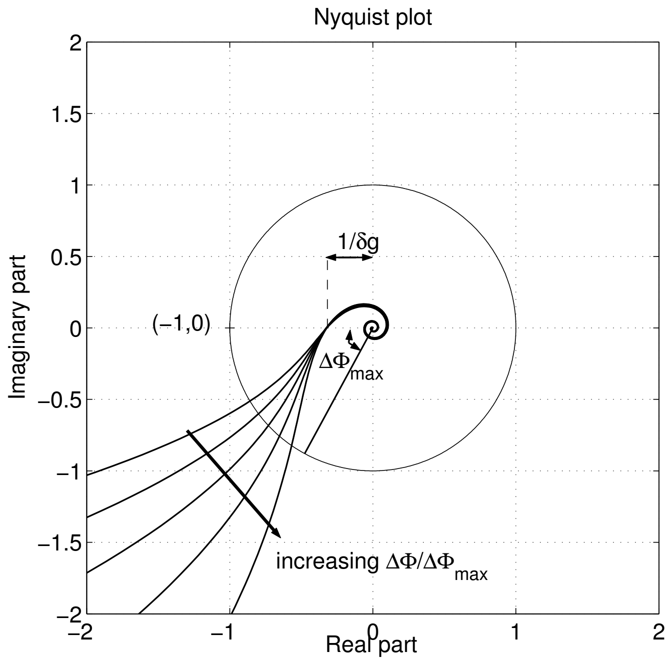

Figure 5: Nyquist plot of the open loop $K_1^*(j\omega^*)G_1^*(j\omega^*)$ for $\Delta G = 10$ dB and different values of phase margin ratio $\Delta\Phi/\Delta\Phi_{\max} \in [0.5, 0.9]$.

dimensionless crossover frequency is about 0.5, which means that perturbations occurring at frequencies higher than $0.5/\tau_d$ rad/s cannot be efficiently attenuated by the controller.
The maximum phase and delay margins are a measure of the maximum achievable robustness of the controlled system for a given gain margin. For example, with $\Delta G = 10$ dB, the maximum phase margin is $\Delta\Phi_{\max} = 65°$. The maximum dimensionless delay margin is $\Delta\tau^*_{\max} = 2.16$, which means that for a phase margin ratio of 0.7, $\Delta\Phi = 0.7\Delta\Phi_{\max} = 43°$, the delay of the canal pool can be increased by 150 % ($0.7 \times 2.16 = 1.5$) without destabilizing the controlled system.

Nyquist plot

The gain and phase margins can also be depicted on a Nyquist plot, where the open-loop $K_1^*(j\omega^*)G_1^*(j\omega^*)$ is plotted in the complex plane for $\omega^* \in [0, +\infty)$. In this plot, the gain and phase margins can be directly evaluated, since they represent a measure of the distance between the open-loop and the critical point $(-1, 0)$. When the open-loop curve crosses this point, the system becomes unstable.
Fig. 5 gives the Nyquist plot of the open-loop of the dimensionless system with different PI controllers having the same gain margin $\Delta G = 10$ dB, and different phase margins. One observes that the tuning method indeed leads to the desired gain margin: the open-loops cross the negative real axis around -0.320. This leads to a gain margin of $\delta g = 1/0.320 = 3.125$, corresponding to a gain margin in dB $\Delta G = 20\log_{10}(3.125) = 9.9$ dB. The phase margin can be chosen between 0 and $\Delta\Phi_{\max}$. Fig. 5 depicts the open-loops $K_1^*G_1^*$ corresponding to controllers with a gain margin of 10 dB and phase margins ratios $\Delta\Phi/\Delta\Phi_{\max}$ between 0.5 and 0.9.

PI Controller Design for Pool 4 of ASCE Test Canal 1

Let us compute the parameters of a PI controller for this canal pool with a gain margin of 10 dB. In this case, the maximum phase margin is $\Delta\Phi_{\max} = 61°$ (Table 2) and with a phase margin ratio of $\Delta\Phi/\Delta\Phi_{\max} = 0.7$, the phase margin is $43°$.
The tuning method gives the following dimensionless PI parameters: $k_p^* = 0.471$ and $T_i^* = 6.03$ (Table 1).

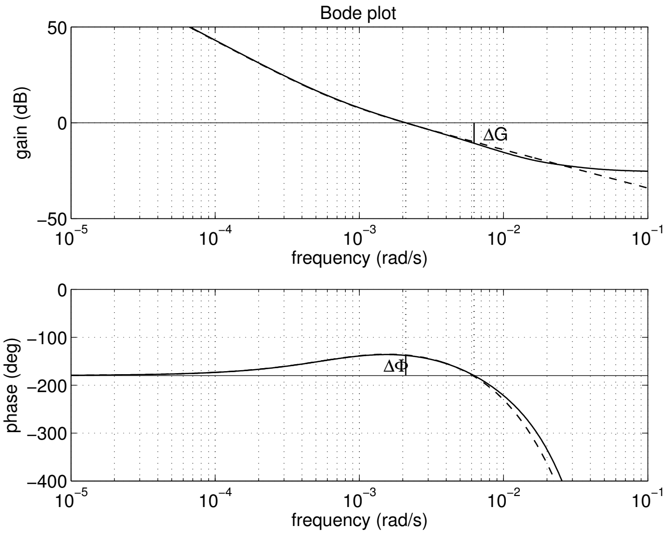

Figure 6: Bode plot of K_1G_1 with the linearized Saint-Venant model (—) and ID model (– –) for pool 4 of ASCE Test Canal 1, high flow conditions.

Using the ID model parameters of the pool and Eqs. (5–6), the dimensional parameters are computed as $k_p = 0.471 \times 546/238 = 1.08$ m^2/s and $T_i = 6.03 \times 238 = 1430$ s.

With those parameters, the real system represented by linearized Saint-Venant equations has a gain margin of 10.7 dB and a phase margin of 42.2°. Fig. 6 depicts the Bode plots of the open loop with the ID model and with the complete linear model obtained from Saint-Venant equations following Litrico and Fromion (2004a). Once again, the approximated frequency response of the open-loop is very close to the real one.

This tuning rule has been tested on many different canals and effectively leads to the desired gain and phase margins.

From Robustness Margins to Time-Domain Performance

Let us now study the influence of the gain and phase margins on the time-domain behavior of the controlled canal pool. This study is based on dimensionless simulation results, with a generic scenario including a step change of the downstream level set-point (y^*_{sp} changes from 0 to 1 at time $t^* = 0$), followed after 50 time units by a step change of the downstream perturbation ($p^* = 1$ at time $t^* = 50$). The simulation is stopped after 100 time units.

The system is simulated on Matlab for different controllers, with different gain and phase margins. The gain margin is expressed in dB (from 6 to 14 dB, which corresponds to absolute gain margins from 2 to 5), and the phase margin is specified by the ratio $\Delta\Phi/\Delta\Phi_{\max}$.

Influence of the Gain Margin ΔG

The choice of a gain margin directly influences the minimum value of $y^*(t^*)$ in response to a unit step perturbation. This is illustrated in Fig. 7, where the time response of the dimensionless controlled system is depicted for different dimensionless PI controllers having the same gain margin $\Delta G = 10$ dB and different values of phase margin. In this case, the maximum dimensionless deviation from equilibrium in response to a unit step perturbation is equal to 2.

The same behavior is observed when the system is simulated for different values of the gain margin ΔG: for a given value of ΔG, the maximum deviation of $y^*(t^*)$ in response to a unit step perturbation is almost the same whatever the chosen phase margin ratio. This is depicted in Fig. 8, where the maximum error on y^* is depicted as a function of ΔG for different values of $\Delta\Phi/\Delta\Phi_{\max}$.

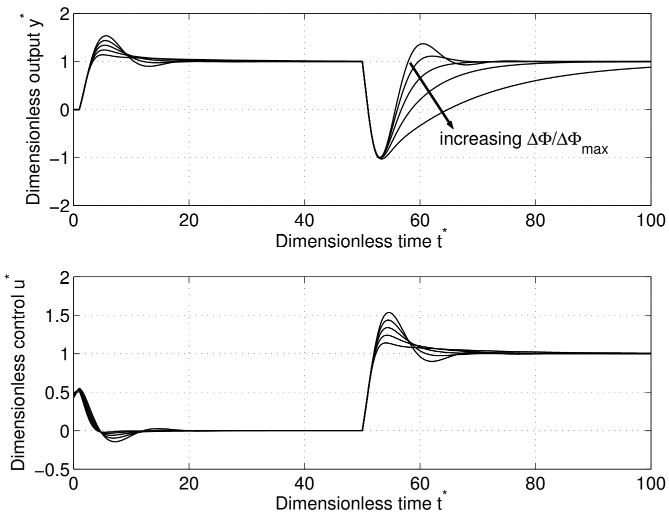

Figure 7: Dimensionless linear simulation for $\Delta G = 10$ dB and different values of the phase margin ratio $\Delta\Phi/\Delta\Phi_{\max} \in [0.5, 0.9]$.

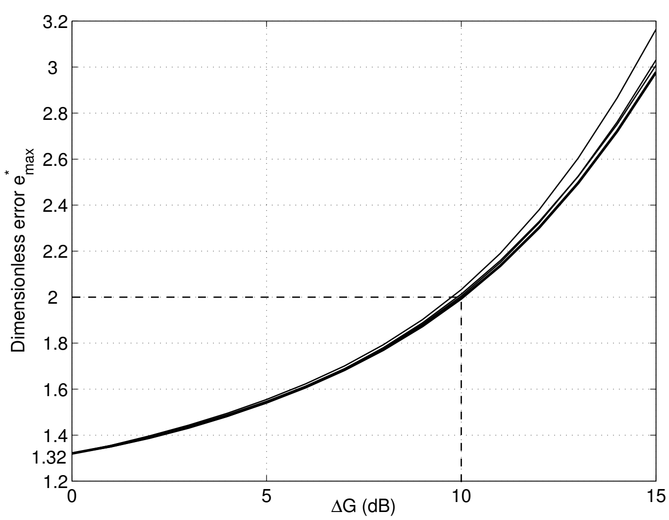

Figure 8: Dimensionless maximum error as a function of ΔG and different values of the phase margin ratio $\Delta\Phi/\Delta\Phi_{\max} \in [0.5, 0.9]$.

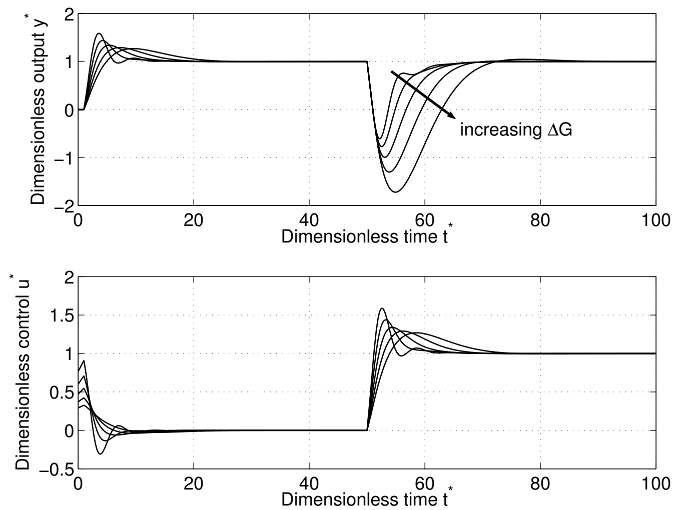

Figure 9: Dimensionless linear simulation for $\Delta\Phi = 0.7\Delta\Phi_{\max}$ and values of the gain margin ΔG from 6 to 14 dB with increment of 2 dB.

The gain margin therefore directly determines the maximum deviation of y^* in response to a unit step perturbation, denoted by $e^*_{\max} = \max|y^* - y^*_{sp}|$. For a canal pool controlled with a distant downstream PI controller with a given gain margin, an unknown step perturbation of Q_p m^3/s will lead to a dimensional error for the downstream water level of:

$$e_{\max} = \frac{e^*_{\max}\tau_d Q_p}{A_d} \tag{16}$$

where $e^*_{\max}$ is given by Fig. 8. For $\Delta G = 10$ dB, one gets $e^*_{\max} = 2$. For higher gain margins, the effect of the phase margin ratio becomes visible, then one can use an average value of $e^*_{\max}$ to determine the gain margin.
Such a result enables us to answer the first point of the design requirements: there is a minimal dimensionless error of 1.32 (for a dimensionless withdrawal of 1), that cannot be reduced using any stable distant downstream PI controller (see Appendix II for details). Therefore, if the required maximum dimensionless error is greater than 1.32, the corresponding gain margin of the controller can be obtained from Fig. 8. If the required maximum dimensionless error is smaller than 1.32, the design specifications cannot be fulfilled with a PI controller.

Influence of the Phase Margin $\Delta\Phi$

Let us now focus on the influence of the phase margin on the time response. For a given gain margin, a smaller phase margin ratio $\Delta\Phi/\Delta\Phi_{\max}$ decreases the time to reach the equilibrium after perturbation leading to a quicker response (see Fig. 7). Fig. 9 depicts the time response of the dimensionless controlled system for a fixed phase margin ratio $\Delta\Phi = 0.7\Delta\Phi_{\max}$ and different values of the gain margin, for the same scenario (step responses to reference level and perturbation). The time responses are qualitatively similar for controllers having the same phase margin ratio.
There are two interesting quantities to study: the time to reach the reference water level at ± 10 % in response to an unknown perturbation and the value of the overshoot in response to a step change of the water level set-point. Usually for irrigation canal control, there are no changes in the water level set-point, but this may happen in some cases. In addition, the overshoot also gives a measure of the sensitivity to sensor noise: if the overshoot is too large, the system tends to be sensitive to noise which affects the water level sensors (Franklin et al., 2002).
The dimensionless time to reach the reference water level after a step perturbation t^*_{10} is depicted in Fig. 10 as a function of the phase and gain margins. With $\Delta\Phi = 0.7\Delta\Phi_{\max}$ and $\Delta G = 10$ dB, one gets

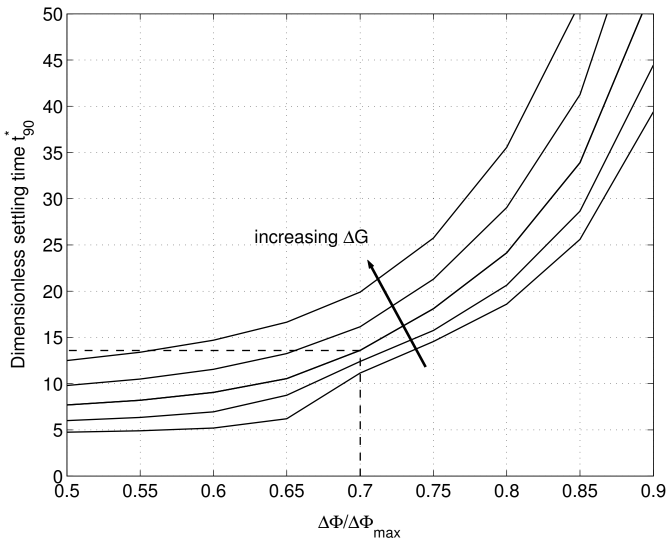

Figure 10: Dimensionless time t^*_{10} as a function of $\Delta\Phi/\Delta\Phi_{\max}$ and values of the gain margin ΔG from 6 to 14 dB with increment of 2 dB.

$t^*_{10} = 13$. This means that the settling time after a perturbation will be about 13 times the time delay for such a controller.
Figs. 8 and 10 enable us to predict the time-domain performance of a given PI controller from the value of its gain and phase margins. But it also provides a way to design controllers based on time-domain design requirements.
As in the case of the maximum error, one may evaluate the minimum time to reach the water level set-point after a step perturbation (see Table 3). This time is obtained by defining a minimum phase margin ratio of 0.5, which is necessary for good performance.

Table 3: Lower bound on t^*_{10} for different gain margins ΔG

ΔG (dB)	1	2	4	6	8	10	12	14	16
t^*_{10}	3.25	3.40	3.90	4.75	6.0	7.70	9.80	12.5	15.9

Fig. 11 gives the value of the overshoot $\max|y^*_{sp} - y^*|$ in the response to a step change of the downstream water level set-point as a function of the phase margin ratio and for different values of the gain margin. The larger the gain margin, the smaller the overshoot, and the same applies for the phase margin ratio.

Application to pool 4 of ASCE Test Canal 1

Let us consider the effect of a small unpredicted perturbation equal to $Q_p = -0.05$ m^3/s, in order to evaluate the validity of the approach in a quasi-linear context. The companion paper will consider more drastic changes in discharge, that are explicitly taken into account in the robust design method.
The findings of the paper enable us to compute the value of the maximum water level deviation from equilibrium corresponding to this withdrawal for the obtained PI controller with 10 dB gain margin.
In this case, the reference water level deviation is $y_r = Q_p \times \tau_d/A_d = 0.05 \times 238/546 = 0.022$ m. The minimum error, obtained for zero gain margin is therefore $1.32 \times y_r = 0.029$ m.
According to Fig. 8, a PI controller with 10 dB gain margin leads to a maximum dimensionless error of 2. Application of Eq. (16) gives the maximum downstream water level deviation from equilibrium $e_{\max} = 2 \times y_r = 0.044$ m.
Fig. 12 compares a full Saint-Venant nonlinear simulation of the controlled system with a linear ID model simulation. The sampling time in both cases is 6 seconds to mimic a continuous time controller Aström and Wittenmark (1990). The nonlinear simulation is done using SIC (Simulation of Irrigation

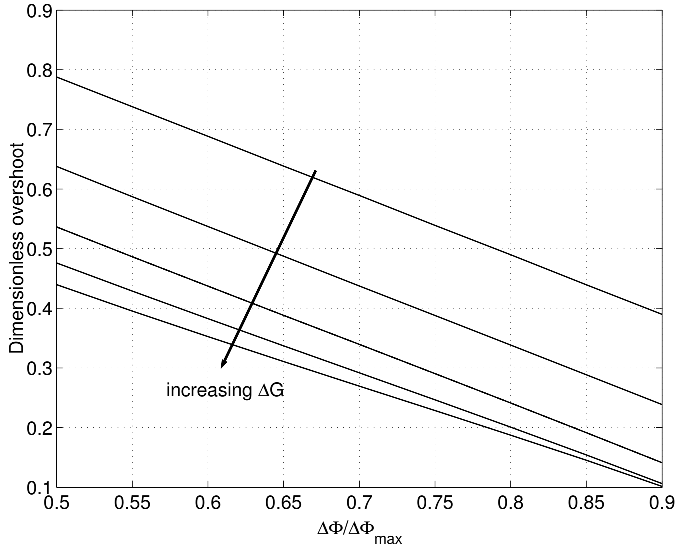

Figure 11: Value of the overshoot for the step of reference downstream level as a function of $\Delta\Phi/\Delta\Phi_{\max}$ and values of the gain margin ΔG from 6 to 14 dB with increment of 2 dB.

Canals), a computer program developed by Cemagref, solving the complete nonlinear Saint-Venant equations with the semi-implicit Preissmann scheme (Malaterre and Baume, 1997).
The maximum deviation from equilibrium in the case of the nonlinear simulation model is 0.047 m, whereas the predicted value is 0.044 m.
With a phase margin ratio of 0.7, the dimensionless time to reach $-0.1 \times y_r$ is equal to 13.6, which corresponds to a dimensional time of $t_{10} = 13.6 \times 238 = 3237$ s. This time is pictured on Fig. 12 by a cross.
Both linear and nonlinear time domain simulations are rather close. The values obtained by the linear prediction have the same order of magnitude as those obtained from nonlinear simulation. In fact, the small mismatch that appears is due more to the simplification of the linear model (use of the ID model instead of a more complex linear model) than to the nonlinearities. An accurate linear model of Saint-Venant equations would give a very close result to the nonlinear simulation. The approach is therefore validated in a quasi-linear context, since it enables to have a good prediction of both the maximum error and the settling time. The nonlinear effects are taken into account in the companion paper by considering various linear models for different regimes.

Conclusion

The paper has presented and validated a methodology to design efficient PI controllers for an irrigation canal pool, based on a simple approximation of the Saint-Venant model.
Tuning rules for distant downstream PI controllers for a canal pool have been derived by specifying gain and phase margins. The link between these robustness margins and the time-domain performance has been emphasized, which gives the engineer the ability to tune the controller according to his specific requirements. These requirements include the maximum error after an unpredicted perturbation has occurred and the time to reach the water level set-point at ± 10 % after the perturbation.
The method enables to predict the time-domain behavior of a canal pool controlled with a PI controller. Since minimum gain and phase margins are necessary, one may analyze whether the design requirements are fulfilled for a given perturbation scenario. This is an important output of the present work.
The tuning method has been applied in simulation on a pool taken from the ASCE test cases. For small perturbations, the simulation on a full nonlinear model is shown to be close to the one obtained with a simple linear ID model.

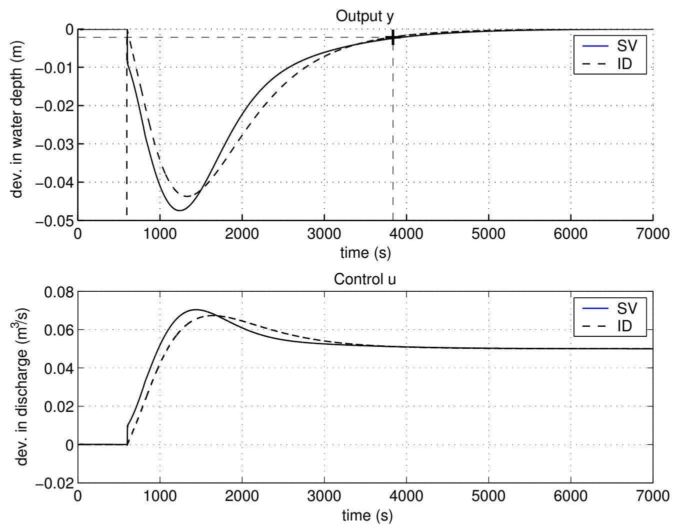

Figure 12: Nonlinear model simulation (SV —) vs linear model simulation (ID – –) of the distant downstream PI controller to a downstream withdrawal, pool 4 of ASCE Test Canal 1, high flow conditions

The above tuning rules apply for a canal pool approximated by a ID model with a given delay and backwater area. However, this model is an approximation of reality and its parameters vary when hydraulic parameters of the pool vary (e.g. the discharge, the friction coefficient, the downstream water elevation, the pool geometry, etc.). As already mentioned in the design specifications, the controller should be robust to take account of these variations. Indeed, the practical problem is that we search for a unique controller for a family of systems (e.g. a canal pool operating at different discharges). This robust controller tuning is examined in the companion paper.

Acknowledgments

The authors acknowledge the financial help of Cemagref and INRA through the collaborative program ASS AQUAE n° 2. We thank our colleagues and especially Jean-Pierre Baume for fruitful discussions on the paper.

Notations

The following symbols are used in this paper:

* = superscript for dimensionless variables;
A_d = downstream backwater area in m^2;
$e_{\max}$ = maximum water level deviation in response to a discharge perturbation;
$G_i(s)$ = transfer functions;
j = complex number $j^2 = -1$;
K_1 = controller transfer function;
k_p = proportional gain in m^2/s;
p = downstream perturbation in m^3/s;
Q_p = perturbation discharge in m^3/s;
Q_r = reference discharge in m^3/s;
q = relative discharge in m^3/s;
s = Laplace variable in s^{-1};
T_i= integral time in s;
t = time in s;

t_{10} = settling time after a step perturbation in s;
u_1 = upstream control variable in m^3/s;
u_2 = downstream control variable in m^3/s;
v=integration variable;
y = relative water elevation in m;
y_r = reference water level deviation in m;
y_{sp} = water level set-point deviation in m;
ΔG = gain margin in dB;
$\Delta\Phi$ = phase margin in degrees;
$\Delta\Phi_{\max}$ = maximum phase margin in degrees;
$\Delta\tau$ = delay margin in s;
$\Delta\tau_{\max}$ = maximum delay margin in s;
δg = absolute gain margin;
ω = frequency in rad/s;
ω_{180} = frequency where the phase of the open-loop equals $-180°$ in rad/s;
ω_c = crossover frequency in rad/s;
τ_d = time-delay for downstream propagation in s;

Appendix I. Impact of Saint-Venant Oscillating Modes on the Frequency Response

The Integrator Delay models well the low frequency behavior of the canal pool. However, there is a discrepancy in high frequencies. The canal transfer function obtained from Saint-Venant equations has a non zero static gain in high frequencies, corresponding to the hydraulic interaction of upstream and downstream propagating waves.
The Integrator Delay Zero (IDZ) model presented in Litrico and Fromion (2004b) models the pool dynamics by an integrator, a delay and a zero, corresponding to a static gain occurring in high frequencies. This model leads to the following approximations of transfer functions G_1 and G_2:

$$G_1(s) \approx \left(\frac{1}{A_d s} + p_{21_\infty}\right) e^{-\tau_d s} \tag{17}$$

$$G_2(s) \approx -\frac{1}{A_d s} - p_{22_\infty} \tag{18}$$

The coefficients of the IDZ model can either be computed analytically for pools with uniform geometry (see Litrico and Fromion (2004b)) or fitted on the complete Saint-Venant numerical frequency response in more complex cases.
Let us examine the impact of the zero of the IDZ model on the frequency response of the system. This zero induced by the constant gain at infinity modifies the gain of the ID model at a frequency denoted ω_m. Using the IDZ model approximation, this frequency is given by

$$\omega_m = \frac{1}{A_d p_{21_\infty}}$$

Let $\omega_{180} = \pi/2\tau_d$ denote the frequency where the phase of the system equals $-180°$. The tuning formulas developed in the paper assume that the ID model is valid up to this frequency ω_{180}.
Therefore, if the frequency ω_m is higher than ω_{180}, then the time-domain performance of the controlled canal pool is linked to the time-delay, and the PI tuning rules provided above can be used directly. On the contrary, if $\omega_m < \omega_{180}$ it is not the delay that limits the performance of the canal, but the oscillating modes. In order to prevent the oscillations to destabilize the controlled system, they should be filtered. This is classically done with a first order low-pass filter, see Schuurmans et al. (1999b); Weyer (2002). However, one should notice that canal pools are usually controlled with discrete-time controllers. In this case, an anti-aliasing filter is implemented to filter signals with a frequency higher than the Nyquist

frequency $\omega_N = \pi/T_s$, with T_s the sampling time. Therefore, if $\omega_N < \omega_m$, there is no need to filter the oscillating modes. The only case where it may be necessary to add a filter to the controller is when $\omega_m < \omega_{180}$ and $\omega_m < \omega_N$. This specific point is not developed in the paper.

Appendix II. Computation of the Minimal Dimensionless Error for a Stable PI Controller

Let us consider the largest proportional controller such that the system remains stable, in order to obtain the minimal error in the downstream water level after a step perturbation. According to the Nyquist stability criterion (Franklin et al., 2002), the largest admissible proportional feedback controller gain is $k_p^* = \pi/2$. Indeed, the stability limit is obtained for k_p^* such that $k_p^* e^{-j\omega^*}/(j\omega^*) = -1$, i.e. for $\omega^* = \pi/2$, and $k_p^* = \omega^*$.

Let us now compute the corresponding error in the downstream water level after a step perturbation for the system controlled with this proportional controller. During a time period less than or equal to the time delay, no water can reach the downstream end of the canal, therefore the water level decreases as it would in open-loop, i.e. $y^*(t^*) = -t^*$. Then, $u_1^*(t) = -k_p^* y^*(t^*)$ for $0 \leq t^* \leq 1$. For $1 \leq t^* \leq 2$, one gets:

$$y^*(t^*) = -t^* + \int_1^{t^*} u_1^*(v-1)dv$$

Thus

$$y^*(t^*) = -t^* + k_p^* \frac{(t^*-1)^2}{2}$$

The time corresponding to the minimum water level is given by $dy^*/dt^* = 0$, which leads to $t^*_{\min} = 1 + 1/k_p^* = 1 + 2/\pi \approx 1.64$ and the corresponding y^* is given by:

$$y^*(t^*_{\min}) = -1 - \frac{1}{\pi} = -1.32$$

References

Aström, K. and Wittenmark, B. (1990). *Computer controlled systems: theory and design.* Prentice-Hall International Editions, London. 544 p.

Aström, K. J. and Hägglund, T. (1995). *PID Controllers: Theory, Design, and Tuning.* Instrument Society of America, Research Triangle Park, NC, second edition. 351 p.

Baume, J.-P., Malaterre, P.-O., and Sau, J. (1999). Tuning of PI to control an irrigation canal using optimization tools. In *Workshop on Modernization of Irrigation Water Delivery Systems*, pages 483–500, Phoenix, Arizona, USA.

Burt, C. M., Mills, R. S., Khalsa, R. D., and Ruiz Carmona, V. (1998). Improved proportional-integral (PI) logic for canal automation. *J. Irrig. Drain. Eng.*, 124(1):53–57.

Clemmens, A. J., Kacerek, T. F., Grawitz, B., and Schuurmans, W. (1998). Test cases for canal control algorithms. *J. Irrig. Drain. Eng.*, 124(1):23–30.

Clemmens, A. J. and Schuurmans, J. (2004). Simple optimal downstream feedback canal controllers: theory. *J. Irrig. Drain. Eng.*, 130(1):26–34.

Franklin, G. F., Powell, J. D., and Emami-Naeini, A. (2002). *Feedback Control of Dynamic Systems.* Prentice Hall, 4th edition.

Litrico, X. and Fromion, V. (2004a). Frequency modeling of open channel flow. *J. Hydraul. Engrg.*, 130(8):806–815.

Litrico, X. and Fromion, V. (2004b). Simplified modelling of irrigation canals for controller design. *J. Irrig. and Drain. Engrg.*, 130(5):373–383.

Malaterre, P.-O. and Baume, J.-P. (1997). SIC 3.0, a simulation model for canal automation design. In *Int. Workshop on the Regulation of Irrigation Canals RIC'97*, Marrakech (Morocco).

Malaterre, P.-O. and Baume, J.-P. (1998). Modeling and regulation of irrigation canal: existing applications and on-going researches (tutorial paper). In *IEEE International Conf. on Systems, Man and Cybernetics (SMC98)*, pages 3850–3855, San Diego, California.

Malaterre, P.-O., Rogers, D. C., and Schuurmans, J. (1998). Classification of canal control algorithms. *J. Irrig. and Drain. Engrg.*, 124(1):3–10.

Schuurmans, J. (1997). *Control of water levels in open-channels.* Ph.D. thesis, ISBN 90-9010995-1, Delft University of Technology.

Schuurmans, J., Clemmens, A. J., Dijkstra, S., Hof, A., and Brouwer, R. (1999a). Modeling of irrigation and drainage canals for controller design. *J. Irrig. Drain. Eng.*, 125(6):338–344.

Schuurmans, J., Hof, A., Dijkstra, S., Bosgra, O. H., and Brouwer, R. (1999b). Simple water level controller for irrigation and drainage canals. *J. Irrig. Drain. Eng.*, 125(4):189–195.

Seatzu, C. (1999). Design and robustness analysis of decentralized constant volume-control for open-channels. *Applied Mathematical Modelling*, 23(6):479–500.

Wahlin, B. T. and Clemmens, A. (2002). Performance of historic downstream canal control algorithms on ASCE test canal 1. *J. Irrig. Drain. Eng.*, 128(6):365–375.

Weyer, E. (2002). Decentralised PI controller of an open water channel. In *15th IFAC World Congress*, Barcelona, Spain.